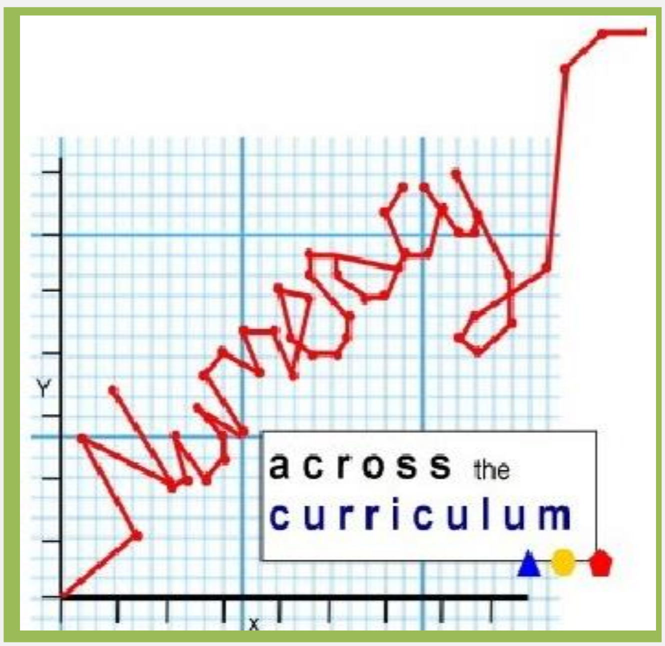
WAGGA WAGGA HIGH SCHOOL

Learning together for the future

Numeracy Across the Curriculum



Numeracy Essentials Guide for Teachers and Parents

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Introduction

Aims of this booklet:

- To enable all teachers and parents to adopt a common approach to numeracy methods across all curriculum areas.

- To enable pupils to more easily recognise the numeracy skills required for their work and will ensure consistency in the methods they will use.

- To enable parents to build confidence in currently taught methods in order to support their child in Numeracy across all curriculum areas.

How to Use this Booklet:

- This highlights the importance of ensuring Numeracy skills are taught through all curriculum areas, together with indicating how teachers should ensure areas of Numeracy are identified in their planning and highlighted to pupils.

- The Numeracy Methods section breaks down the key skills into the 4 core areas of Numeracy. Examples are used to show the current widely used methods. In most cases there is more than one method. Teaching staff and Parents should be guided by the method which the child feels most confident with, rather than insisting on one particular strategy.

- This guide is not supposed to be exhaustive, but as a guide to current methodology. All topics are not covered, if any further information or support is required, please contact Mr. Chittick or one of the Mathematics teachers at WWHS.

- In the electronic version, click the contents page section using **CTRL and CLICK**, this will take you to the relevant section.

COMMON METHODOLOGY

Place Value

- Every number can be 'partitioned' into its component parts

e.g. 2,465.12 = 2000 + 400 + 60 + 5 + 0.1 + 0.02

The Units column is the single digits, followed to the left by tens, hundreds, thousands, ten thousands, hundred thousands, millions etc.

0.1 = 1 tenth, tenths are the first column after the decimal point. There are ten tenths in a whole.
0.01 = 1 hundredth. There are ten hundredths in a tenth.

When dealing with numbers, always ensure the columns are lined up on top of each other including the decimal point which should be on top of each other.

e.g.	123.49	NOT	123.49
+	36.4		36.49

Square numbers

Square numbers are the result of multiplying a number by itself.
e.g. 1x1 = 1, 2x2=2
These are written using powers e.g. $4 \times 4 = 4^2$

They can be used in many areas of Mathematics including finding Area of circles.

Estimation and rounding

We can use rounded numbers to give us an approximation. We can then use this to estimate the answer to a calculation. This allows us to check that our answer is sensible. We generally round using the first non-zero digit i.e. 1st significant figure.

Rounding Whole Numbers

Numbers can be rounded to give an approximation, either up or down. In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

46 753 = 47 000 to the nearest thousand

Rounding to Decimal Places

Example 1 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

1.57359
= 1.57 to 2 decimal places

Rounding to Significant Figures

Numbers can also be rounded to a given number of significant figures. Start with the first non-zero number. This is the 1st significant figure.

Example 2 Round 0.15273 to 2 significant figures

The first significant figure is 1 in the tenths place
The second significant figure is 5 in the hundredths place

0 . 1 5 2 7 3

1st 2nd

We then look at the next number and decide whether to round the 5 up or keep it the same. It is 2 so we keep the 5 the same

= <u>0 . 1 5 to 2 significant figures</u>

Operations and Calculations

Addition and Subtraction, Multiplication and Division,

Addition

Mental strategies – There are a number of strategies to complete mentally

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74 then 74 + 7 = 81

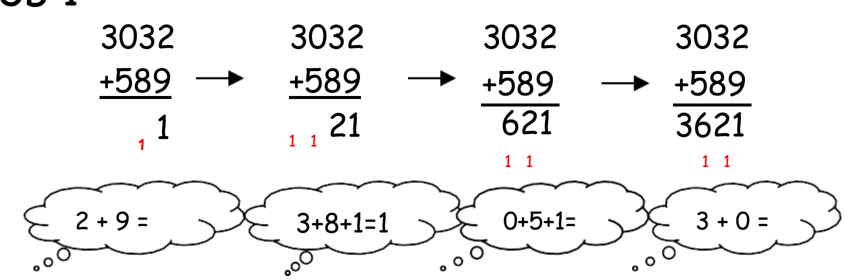
This can also be written on a number line, adding 20 to 54, then 7 to 74.

Written Method

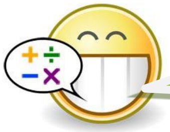
When adding numbers, ensure that the numbers are **lined up** according to place value. Start at right hand side, write down units, carry any tens as 1.

Example I spend \$3032 a year on my car loan. My insurance is \$589. How much is this in total?

METHOD 1



Subtraction



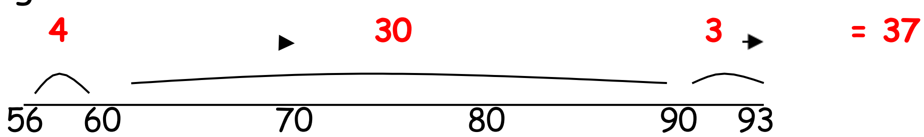
Traditional Column Method can be used, but 'Number Line' Method can be linked to Addition, to also complete Subtraction. This links with Partitioning too!

Mental/Written Strategies

Example Calculate 93 - 56

Method 1 Counting on a Number line

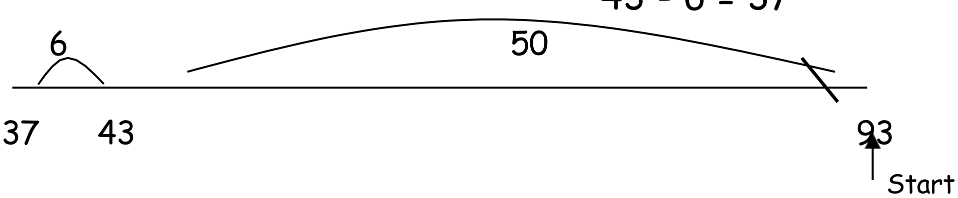
Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 93 - 50 = 43

43 - 6 = 37



Column Method

Example 1 4590 - 386

Example 2 Subtract 692 from 14597

$$\begin{array}{r} 4590 \\ -\ 386 \\ \hline 4204 \end{array}$$

8 1

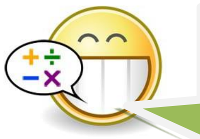
$$\begin{array}{r} 14597 \\ -\ 692 \\ \hline 13905 \end{array}$$

3 1

Multiplication of Whole Numbers

The Times tables up to 12's should be known. These can be used to find any other multiplication sum.

Mental Strategies



You should use the times tables up to 12 x, to help you answer harder questions

Example Find 39 x 6

Method 1

30 x 6 = 180

9 x 6 = 54

180 + 54

Method 2

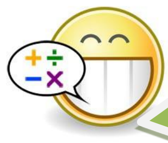
40 x 6 =240

40 is 1 too many so take away

240 - 6 = 234

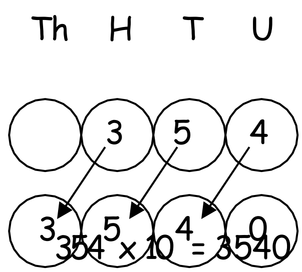
Multiplication

Multiplying by multiples of 10 and 100

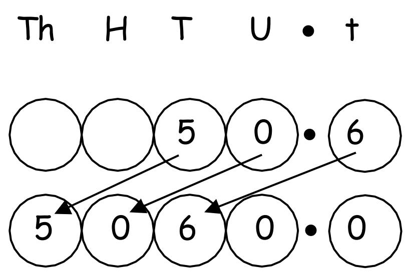


To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



354 x 10 = 3540



50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

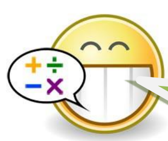
so 35 x 30 **= 1050**

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 **= 261600**



We can also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2

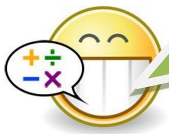
so 2.36 x 20 = 47.2

(b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920

so 38.4 x 50 = 1920

Multiplying larger numbers



There are a number of methods including mental methods like those above. The most commonly taught method is now the grid method. If a pupil is confident at column multiplication, and is always accurate, they should continue to use this method. If mistakes occur, they should try grid method.

Example

There are 35 seats in a row, and 37 rows of seats. Work out if there are enough seats for 1100 people, or will more rows need to be added?

Method 1

Grid Multiplication – This is now the most consistently used method at Secondary level. It uses the smaller multiples to build up larger multiplication sums.

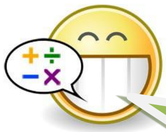
X	30	7
30	=30 x 30 =900	= 7 x 30 =210
5	=30 x 5 =150	=7 x 5 = 35

= 900 +210
= 1110
= 150+35
= 185

1110 + 185 = 1295

- ✓ Partition the numbers into tens and units.
- ✓ Multiply the values 'on the edges',
- ✓ Add up the boxes.

Division



Division is the opposite of multiplication. You should be able to divide by a multiple of 10 or 100 by moving the numbers opposite to that a single digit without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between

8 classes. How many pupils are in each class?

$$8 \,\overline{\big)\, 1\;9^{3}2}\quad \text{answer: } 2\;4$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$3 \,\overline{\big)\, 4\,.\,^{1}7^{2}4}\quad \text{answer: } 1\,.\,5\;8$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$8 \,\overline{\big)\, 2\,.\,^{2}2^{6}0^{4}0}\quad \text{answer: } 0\,.\,2\;7\;5$$

Each glass contains **<u>0.275 litres</u>**

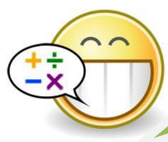
If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Order of Calculation (BIDMAS)

What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is **42**.



Calculations which have more than one operation(i.e. x, + - or ÷) need to be done in a **standard** order. The order can be remembered **BIDMAS**.

The rule means Brackets should be done first.

(B)rackets
(I)ndices
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Scientific calculators use this rule automatically, some basic calculators may not, so take care in their use.

Example 1 $15 - 12 \div 6$ BIDMAS tells us to divide first

$= 15 - 2$

$= 13$

Example 2 $(9 + 5) \times 6$ BIDMAS tells us to work out the brackets first

$= 14 \times 6$

$= 84$

Example 3 $18 + 6 \div (5-2)$ **Brackets first**

$= 18 + 6 \div 3$ Then divide

$= 18 + 2$ Now add

$= 20$

<u>Negative Numbers</u>:

Adding a negative number is the same as subtracting

Subtracting a negative number is the same as adding.

Using a number line:

To ADD count to the right.

To SUBTRACT count to the left.

Examples:

1. -2 +3

Start at -2
Move 3 places to the right

-5 -4 -3 -2 -1 0 1 2 3 4 5

<u>-2 +3 =1</u>

2. 2+(-5)

=2 -5
Start at 2
Move 5 places to the left

-5 -4 -3 -2 -1 0 1 2 3 4 5

<u>2 +(-5) = 3</u>

3. 6 - 10

Start at 6
Move 10 places to the left

-5 -4 -3 -2 -1 0 1 2 3 4 5
6

6 -10 = -4

4. -4 -(-5)
= -4 +5
Start at -4
Move 5 places to the right

-5 -4 -3 -2 -1 0 1 2 3 4 5

-4 -(-5) = 1

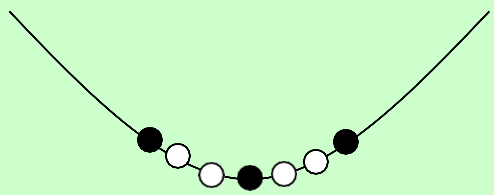
FRACTIONS

Fractions are used to give a proportion of another value or to state how much of a total something is. For example $\frac{1}{4}$ of my salary goes on my mortgage.

Understanding Fractions

Example

A necklace is made from black and white beads.

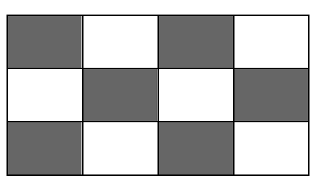


What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

What fraction of the flag is shaded?



6 out of 12 squares are shaded.

So $\frac{6}{12}$ of the flag is shaded. (6 twelfths)

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions

Simplifying Fractions

Equivalent fractions can be simplified as shown below:

The top of a fraction is called the **numerator**, the bottom is called the **denominator**. Both must be whole numbers. To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the **same number.**

Example 1

(a)

$$\frac{20}{25} \underset{\div 5}{\overset{\div 5}{=}} \frac{4}{5}$$

(b)

$$\frac{16}{24} \underset{\div 8}{\overset{\div 8}{=}} \frac{2}{3}$$

This can be done again and again until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in it's simplest form.

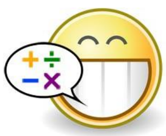
Think of a pizza, 2/3 or pizza is the same as 4/6 of a pizza, only that the slices are bigger or smaller!

Example 2

Simplify $\frac{72}{84}$

$$\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7} \text{ (simplest form)}$$

Calculating Fractions of a Quantity



Fractions share amounts into equal parts.

So to find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of \$150

$\frac{1}{5}$ of \$150 = \$150 ÷ 5 = **<u>\$30</u>**

To find a unit fraction (e.g. $\frac{1}{4}$) divide by the bottom number.

Example 2 Find $\frac{3}{4}$ of 48

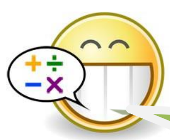
$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

To find any other fraction, divide by the bottom and then multiply by the top

so $\frac{3}{4}$ of 48 = 3 x 12 = **<u>36</u>**

<u>Percentages</u>

Percentage means 'out of 100'. We divide or multiply to make any value out of 100 to write as a percent. They are widely used to give a way of comparing one value out of another. They can be used by shops(sales & discounts), banks (interest rates), the government(tax rates)



The key percentage building blocks can be used to 'build up' any percentage. They are 100% (all of the amount), 50%, 25%, 10%, 5% and 1%. It is vital to know these to get any harder percentage.

Building Blocks

To get any of the building blocks, divide the amount by the following:

100% - All of the amount you start with

50% - divide by 2

25% - divide by 4 or find 50% and divide by 2

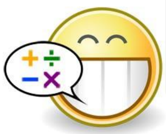
10% - divide by 10

1% - divide by 100.

Some people find using the fraction equivalent easier if they understand, e.g.

$$25\% \text{ of } \$640 = \frac{1}{4} \text{ of } \$640 = \$640 \div 4 = \underline{\mathbf{\$160}}$$

Finding Percentages



Real Life Link: Percentages are used in a variety of places in real life such as Sales in shops, tax on wages, interest on loans, mortgages and bank accounts

Non- Calculator Methods

Example *An Xbox game decreases by 30% from $45. How much will I save?*

Step 1) 'Build the percentage' - **30%= 10% + 10% + 10%**

Step 2) Find the percentages. **10% of $45 = 45 ÷ 10**

= $4.50

(As there are 10 lots of 10% in 100%).

Step 3) **Add the amounts together**. $4.50+$4.50+$4.50 = **$13.50**

So **30% of $45 = $13.50**

Example 2 *A $1,200 holiday to Disneyland has a 6% saving for 1 week only, how much will I save?*

Step 1) 'Build the percentage' - 6%= 5% + 1%

Step 2) Find the percentages. 10% of $1,200 = 1200 ÷ 10

= $120

5% of $1,200= 120 ÷ 2 (Half of 10%)

= $60

1% of $1,200 = 1200 ÷ 100 = $12

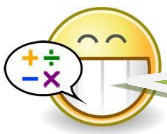
(as there are 100 lots of 1% in 100%)

Step 3) Add the amounts together. $60 + $12 = $72

So **6% of $1200 = $72**

Percentages

Expressing something as a percentage



To find a number as a percentage of another number, first make a fraction, this can then be expressed as a percentage by finding that fraction of 100%.

Example 1 There are 30 pupils in Class 3M. 18 are girls. What percentage of Class 3A3 are girls?

$$\frac{18}{30} = 18 \div 30 = 0.6 = 60\%$$

OR $$\frac{18}{30} \times 100\% = \frac{18 \times 100\%}{30} = \frac{1800}{30} = 60\%$$

60% of 3A3 are girls

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

Score = $\frac{36}{44}$ = 36 ÷ 44 = 0.81818… = 81.818..% = **82% (rounded)**

OR $\frac{36}{44} \times 100\% = \frac{36 \times 100\%}{44} = \frac{3600}{44}$ = 81.818..% = **82% (rounded)**

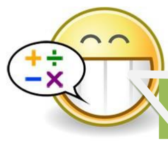
Example 3 In class 2K, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

Total number of pupils = 14 + 6 + 3 + 2 = 25

6 out of 25 were blonde, so,

$\frac{6}{25}$ = 6 ÷ 25 = 0.24 = **24% were blonde**

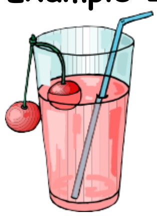
Ratio



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.
The ratio of red : blue : green is 5 : 7 : 8

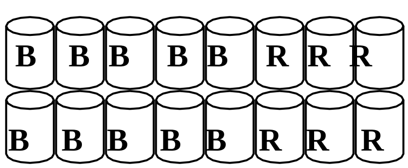
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue:Red =10:6
=5:3

To simplify a ratio, divide each figure in the ratio by the highest number that goes into both numbers.

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= **2:3**

Divide each figure by 2

(b) 24:36
= **2:3**

Divide each figure by 12

(c) 6:3:12
= **2:1:4**

Divide each figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement.
Write
the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
15	**10**

×5 ×5

So the chocolate bar will contain 10g of nuts.

Sharing in a given ratio

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 **Add up the numbers to find the total number of parts**

3 + 2 = 5

Step 2 **Divide the total by this number to find the value of each part**

90 ÷ 5 = £18

Step 3 **Multiply each figure by the value of each part**

3 x £18 = £54
2 x £18 = £36

Step 4 **Check that the total is correct**

£54 + £36 = £90

Lauren received £54 and Sean received £36

Money & Decimal Places

All calculations of money need to be written down to 2 decimal places (two numbers after the decimal point) This could mean that we need to round numbers:

Example 1 Round $1.525 to 2 decimal places

The second number after the decimal point is a 2 - the check digit (the third number after the decimal point) is a 5, so round up.

1.525

= 1.53 to 2 decimal places

We may also need to put in zeros to show our answers to 2 decimal places:

Example 2 Calculate the total cost of the following items

Pencil	20¢
Pen	40¢
Rubber	30¢
Ruler	75¢
Sharpener	25¢

Total cost = 190¢

= $1.90 to 2 decimal places

<u>% Extra Free</u>

Example 1:

A cereal packet usually contains 750g of cereal. There is a special offer packet which contains 25% extra free. How much cereal is in the special offer packet?

Calculate the number of extra grams:

25% of 750g = $\frac{1}{4}$ of 750g

$\frac{1}{2}$ of 750g = 375g

$\frac{1}{4}$ of 750g = 187.5g

Add this to the original number of grams in the packet:

750g + 187.5g = 937.5g

PROBLEM SOLVING

Buy One Get One Free

This offer is usually used when retailers want to clear a large number of items quickly. They are effectively reducing the price of goods by half whilst ensuring that you buy two items at a time.

This offer is only a saving if you would normally use the two items before the goods would be out of date.
If you usually buy one chocolate cake and you get one free, you haven't made a saving you, just have an extra cake. However, if you usually buy two cakes you have made a saving.

Three for the Price of Two

This is similar to the above offer. The retailers are effectively reducing the cost to two thirds of the original price. This offer is only a saving if you would normally use the three items before the goods would be out of date.

Which offer is the best value?

To work this out we need to work out the 'price per one' of something. This can be 100g, 1kg, 1 unit etc. The quantity or amount of each product needs to be the same for a comparison.

Look at the following special offers.

'Swarbricks'	Brown's Bread	Wheaty Bake
600g	800g	790g
78¢ per loaf	$1.20	98¢
3 for 2	20% extra free	10% discount

a) Which offers the best value for money per gram of bread without the special offer?

Swarbricks: 78¢ ÷ 600g
= 0.13p per gram

Brown's Bread 120¢ ÷ 800g
= 0.15¢ per gram

Wheaty Bake 98¢ ÷ 790g
= 0.12¢ per gram

Wheaty Bake is the best value for money at 0.12¢ per gram

b) Which offers the best value for money per gram of bread with the special offer?

Swarbricks: 3 for the price of 2

Cost = 2 x 78
= 156¢

Grams = 3 x 600g
= 1800g

Cost per gram = 156¢ ÷ 1800g
= 0.09¢ per gram

Brown's Bread 20% extra free

Cost = 120¢

Grams = 800g + (20% of 800g)

= 800g + 160g

= 960g

Cost per gram = 120¢ ÷ 960g

= <u>0.13¢ per gram</u>

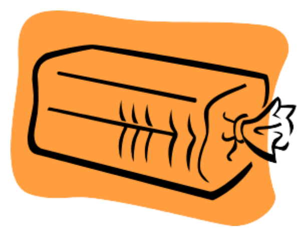
Brown's Bread: 0% extra free

Cost = 98¢ – (10% of 98¢)

= 98¢ – 9.8¢

= 88.2¢

Grams = 790g



Cost per gram = 88¢ ÷ 790g

= <u>0.11¢ per gram</u>

The Swarbrick's special offer is the best value for money if you would normally use 3 loaves of bread before the bread went stale.

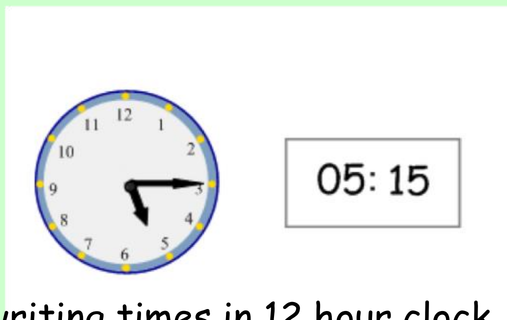
Shape, Space and Measures Time

TIME

Time may be expressed in 12 or 24 hour

12-hour clock

Time can be displayed on a clock face, or digital clock.

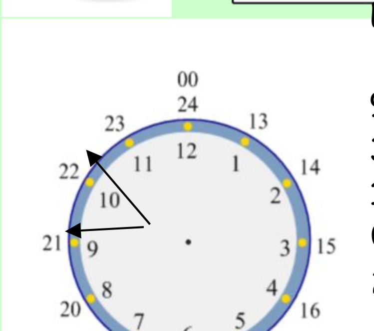


These clocks both show fifteen minutes past five, or quarter past

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock

In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.



Examples

9.55 am → 09 55 hours
3.35 pm → 15 35 hours
12.20 am → 00 20 hours
02 16 hours → 2.16 am
20 45 hours → 8.45 pm

Time Periods

It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
52 weeks
12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Interpreting Timetables

Destination	Time	Time	Time	Time	Time	Time	Time	Time	Time
Thurso Business Park	0645	0745	0905	1005	1105	1205	1305	1405	1505
Olrig Street Job Centre	0650	0750	0910	1010	1110	1210	1310	1410	1510
Halkirk Sinclair Street	0705	0805	0925	1025	1125	1225	1325	1425	1525
Watten Post Office	0718	0818	0938	1038	1138	1238	1338	1438	1538
Haster Fountain Cottages	0725	0825	0945	1045	1145	1245	1345	1445	1545
Wick Somerfield bus terminal	0730	0830	0950	1050	1150	1250	1350	1450	1550
Wick Business park	0735	0835	0955	1055	1155	1255	1355	1455	1555
Wick Tesco Store	0736	0836	0956	1056	1156	1256	1356	1456	1556
Wick Airport Terminal	0741	0841	1001	1101	1201	1301	1301	1401	1601

Examples of Questions:

a) I want to be at Wick Airport by 2.30pm. What time must I catch the bus at Olrig Street Job Centre?

2.30pm is shown as 1430 h on the timetable
The most suitable bus arrives at Wick Airport at 1401
This leaves Olrig Street Job Centre at **1310 h**

b) The 0745 bus from Thurso Business Park is running 6 minutes late. What time does it reach Wick Tesco Store?

Add 6 minutes to the arrival time at Wick Tesco Store

This is 0836 h. <u>It arrives at **0842 h**</u>

How long does the first bus journey from Halkirk to Wick Business Park take?

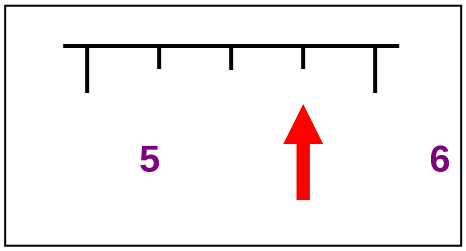
The bus leaves Halkirk at 0705 h and arrives at Wick Business Park at 0735 h.

The journey time is **<u>30 minutes.</u>**

Measurement

Reading scales

Scale 1



In this scale the difference between 5 and 6 is 1, and the space has been divided into 4, so each division represents 1 ÷ 4 = 0.25.

The arrow is pointing to 5 + 0.25 + 0.25 + 0.25 = 5.75

Scale 2 - a speedometer

The difference between 50 and 60 is 10 and the space has been divided into 2, so each division represents 10 ÷ 2 = 5.
The arrow is pointing to 50 + 5 = 55

Converting between units

The table shows some of the most common equivalences between different units of measure. Make sure you know these **conversions**.

Length	Weight	Capacity
	1 tonne = 1000kg	
1 km = 1000m	1kg = 1000g	
1m = 100cm = 1000mm	1g = 1000mg	1l = 100cl = 1000ml
1cm = 10mm		1cl = 10ml

If converting from a larger unit (eg m) to a smaller unit (eg cm), check what number of smaller units are needed to make 1 larger unit, then multiply that number with the relevant number of the larger units.

If converting from a smaller unit (eg cm) to a larger unit (eg m), check what number of smaller units are needed to make 1 larger unit, then divide that number into the relevant number of the larger units.

Remember: To convert from a larger unit to a smaller one, **multiply**.
To convert from a smaller unit to a larger one, **divide**.

Worked example

We know that 1m = 100cm

So, to convert from m to cm we multiply by 100, and to convert from cm to m we divide by 100.

Eg: 3.2m = **320cm** ($3.2 \times 100 = 320$)
400cm = **4m** ($400 \div 100 = 4$)

Metric and imperial units

Imperial measures are old-fashioned units of measure. These days we have mostly replaced them with **metric units**, but despite our efforts to 'turn metric', we **still use many imperial units in our everyday lives.** It is therefore important that we are able to calculate **rough equivalents** between **metric and imperial units**.

Here are some conversions that you will need to know:

1 inch is about 2.5cm

1 foot is about 30cm

- - - - - -

1kg is about 2.2 pounds

8km is about 5 miles

(1km is about 5/8 mile, and 1 mile is about 8/5km)

Worked example

We know that 1 mile is about 1.6 km.

To convert from miles to km, we multiply by 1.6.

To convert from km to miles, we divide by 1.6.

E.g. 20 litres = 32 km (20 x 1.6 = 32)

80 km/hr = 50 mph (80 ÷ 1.6 = 50)

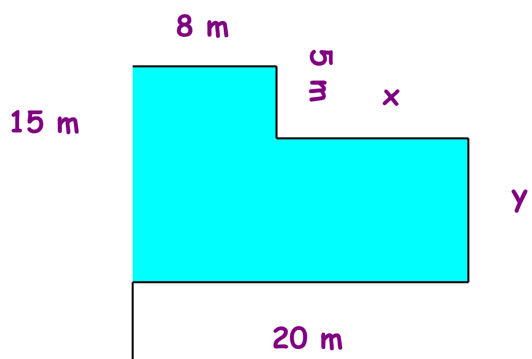
Perimeter (always measured in cm,mm,m,km,ft,in)

The perimeter of a shape is the length of its boundary or outside edges.

Think of a football pitch, If I walk around the edge of the pitch, the distance I walk is the perimeter of the field.

Example question

A plan of a play area is shown below:



a) Calculate the length of x and y

The length of the play area is 20m, so x = 20 - 8 =12m.The width of the play area is 15m, so y = 15 - 5 = 10m.

b) Calculate the perimeter of the play area.

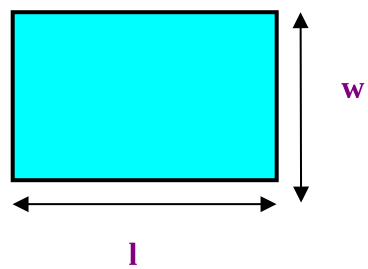
Perimeter = 20 + 15 +8 + 5 + 12 + 10

= 70 m

Area (always measured in cm^2, mm^2, m^2, km^2, ft^2, in^2)

Area of a rectangle

Area = l x w

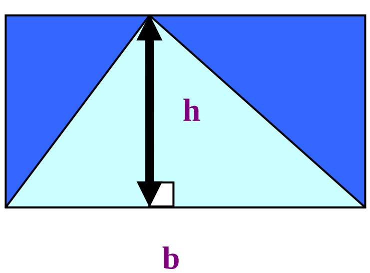


The area of a rectangle is its length multiplied by its width.

The formula is: **area = length x width**

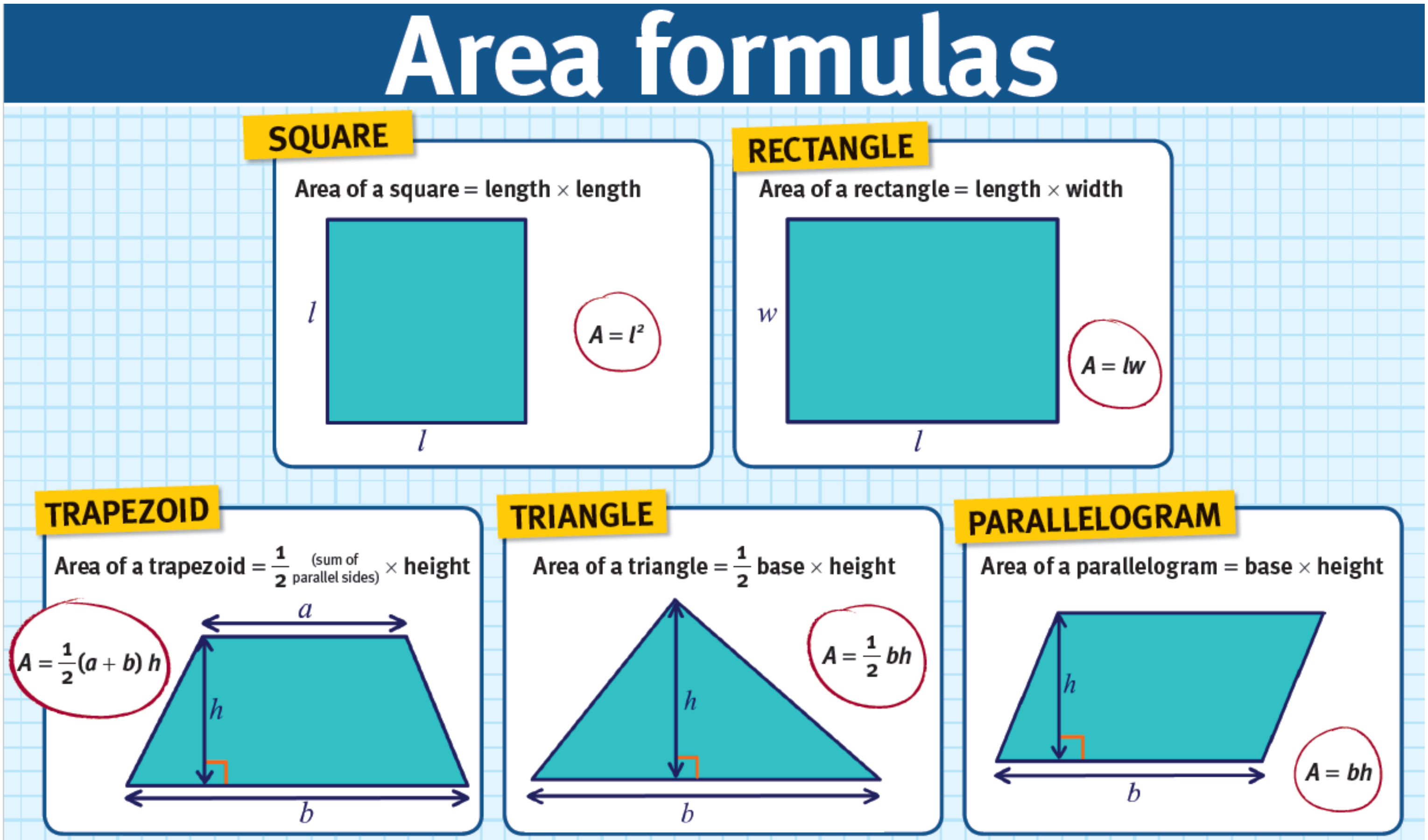
Area of a triangle

Look at the triangle below:

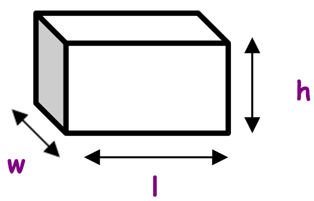


If you multiplied the base by the **perpendicular(at 90° to)** height, you would obtain the area of a rectangle. The area of the triangle is **half the area of the rectangle.** So to find the area of a triangle, we multiply the **base by the perpendicular height and divide by two.** The formula is:

Area $=\frac{1}{2}$ x base x height


Area formulas
SQUARE
Area of a square = length × length
l
l
A = l²
RECTANGLE
Area of a rectangle = length × width
w
l
A = lw
TRAPEZOID
Area of a trapezoid = 1/2 (sum of parallel sides) × height
a
h
b
A = 1/2 (a + b) h
TRIANGLE
Area of a triangle = 1/2 base × height
h
b
A = 1/2 bh
PARALLELOGRAM
Area of a parallelogram = base × height
h
b
A = bh

Volume (always measured in cm^2, mm^2, m^2, km^2

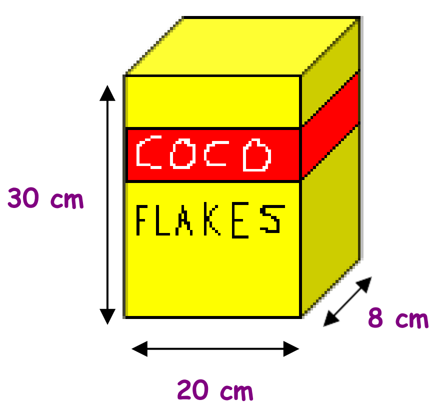


To find the volume of a cuboid we find the front (cross-sectional) area and multiply by its depth or width.

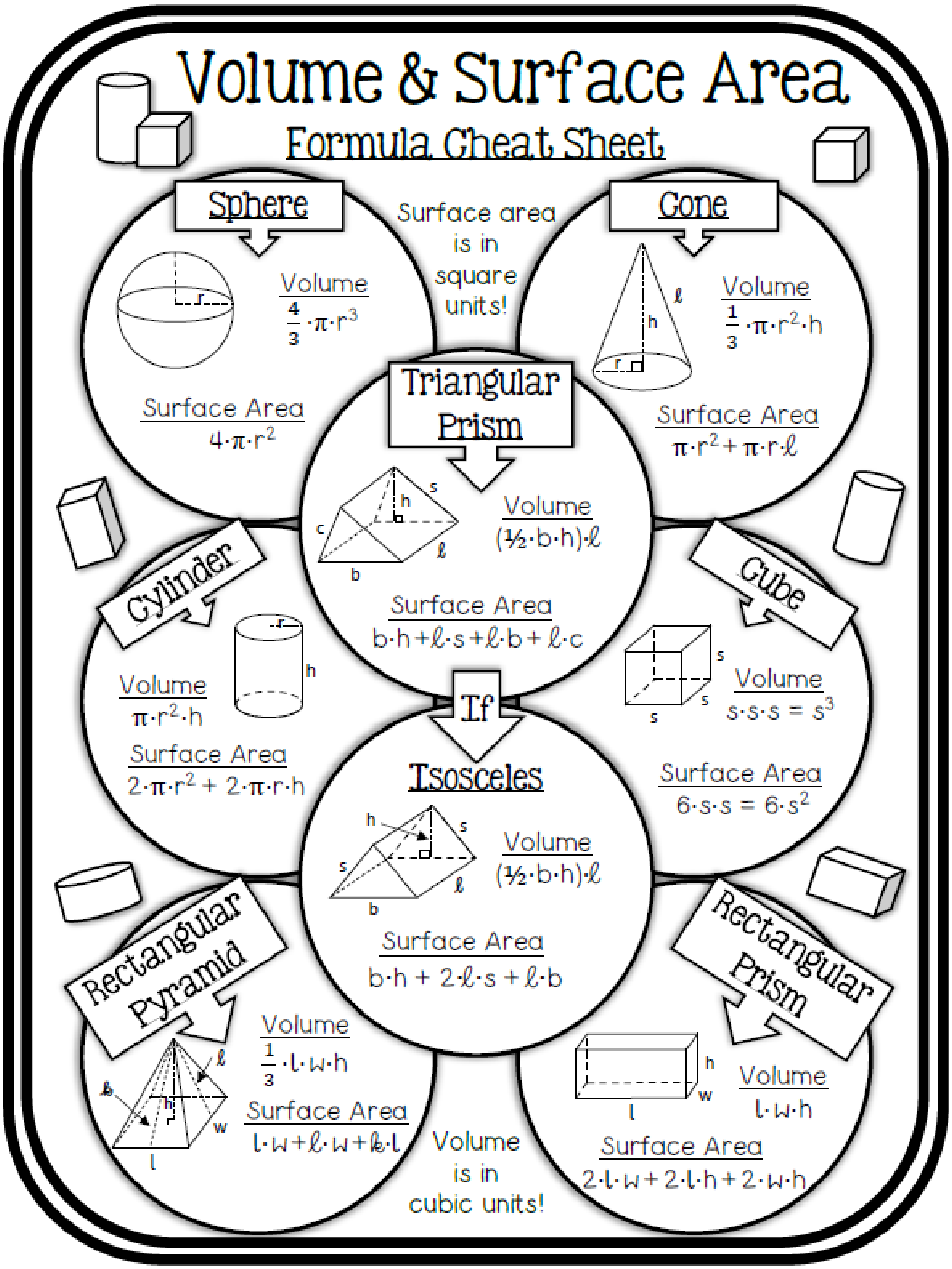
For example:

The volume of this cereal packet is 20 x 30 = 600 cm^2

Then 600 x 8 = **4800cm^3**



Volume can also be measured in Litres. 1000cm^3 = 1 Litre



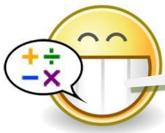
Volume & Surface Area
Formula Cheat Sheet
Sphere
Volume
$\frac{4}{3} \cdot \pi \cdot r^3$
Surface Area
$4 \cdot \pi \cdot r^2$
Surface area is in square units!
Cone
Volume
$\frac{1}{3} \cdot \pi \cdot r^2 \cdot h$
Surface Area
$\pi \cdot r^2 + \pi \cdot r \cdot \ell$
Triangular Prism
Volume
$(½ \cdot b \cdot h) \cdot \ell$
Surface Area
$b \cdot h + \ell \cdot s + \ell \cdot b + \ell \cdot c$
Cylinder
Volume
$\pi \cdot r^2 \cdot h$
Surface Area
$2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h$
Cube
Volume
$s \cdot s \cdot s = s^3$
Surface Area
$6 \cdot s \cdot s = 6 \cdot s^2$
If
Isosceles
Volume
$(½ \cdot b \cdot h) \cdot \ell$
Surface Area
$b \cdot h + 2 \cdot \ell \cdot s + \ell \cdot b$
Rectangular Pyramid
Volume
$\frac{1}{3} \cdot l \cdot w \cdot h$
Surface Area
$l \cdot w + \ell \cdot w + \ell \cdot l$
Volume is in cubic units!
Rectangular Prism
Volume
$l \cdot w \cdot h$
Surface Area
$2 \cdot l \cdot w + 2 \cdot l \cdot h + 2 \cdot w \cdot h$

Statistics

10 Rules for drawing or plotting a graph

1. I shall always put a title on my graph.
2. I shall always think about which type of graph is best to use.
3. I shall always use a pencil and ruler to draw my axes.
4. I shall always try to fill my graph paper with my graph by choosing a suitable scale.
5. I shall always put the dependent variable (one that we measure or observe) on the y axis.
6. I shall always put the independent variable (the one we change) on the x axis.
7. I shall always label both axes
8. I shall always put the units on my axes
9. I shall always plot my points accurately using crosses.
10. I shall always draw a smooth curve or a straight line (with a ruler) where appropriate.

Data Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. We group large amounts of data into group or intervals.

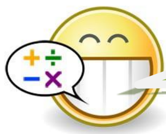
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	||	2
21 - 25	~~||||~~ ||	7
26 - 30	~~||||~~ ||||	9
31 - 35	~~||||~~	5
36 - 40	|||	3
41 - 45	||	2
46 - 50	||	2

Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

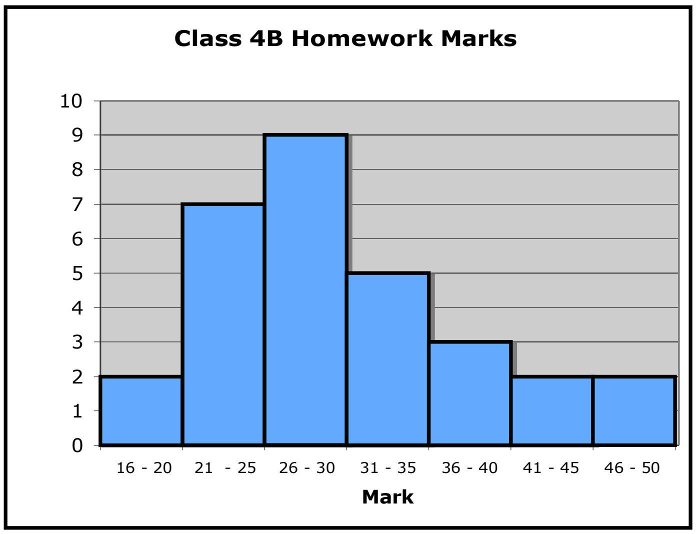
Bar Graphs



Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency.

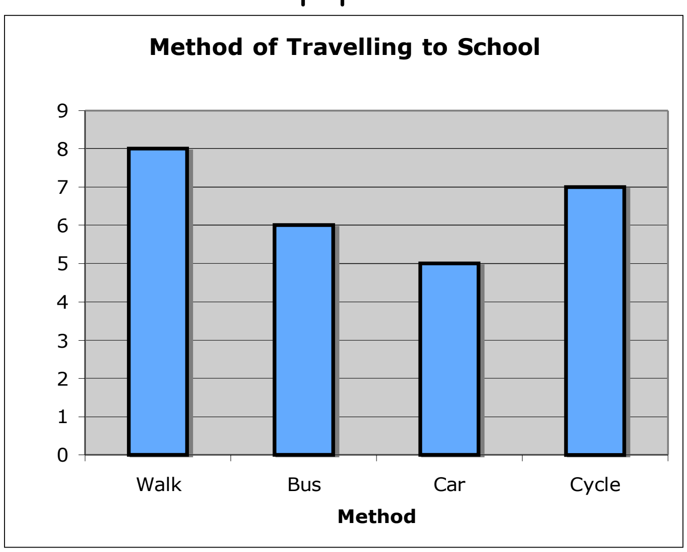
Example 1 The **frequency diagram** graph below shows the homework marks for Class 4B.

Number of Pupils



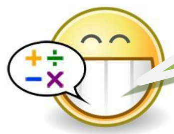
Example 2 A Bar chart to show how pupils travel to school

Number of Pupils



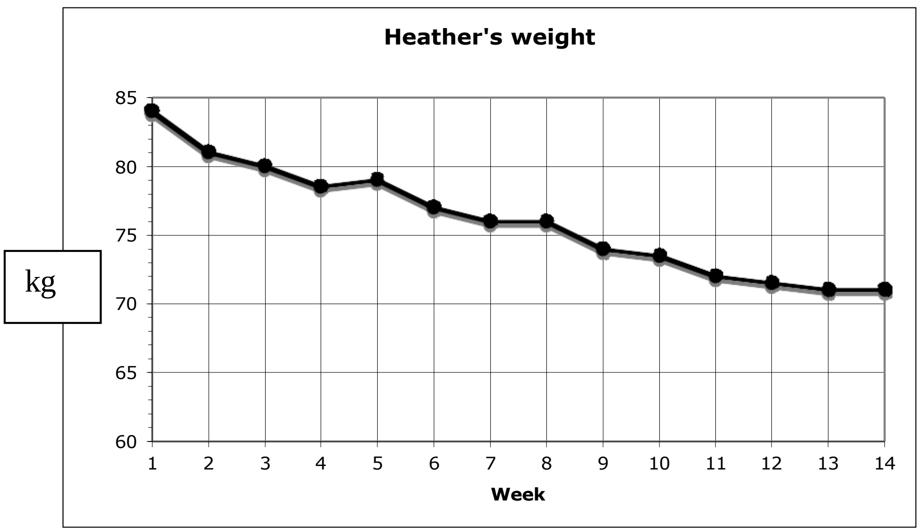
NOTICE this bar chart has gaps between as they are categories not groups.Continuous data(can take any value) is put into a frequency diagram, which has NO gaps.

Line Graphs



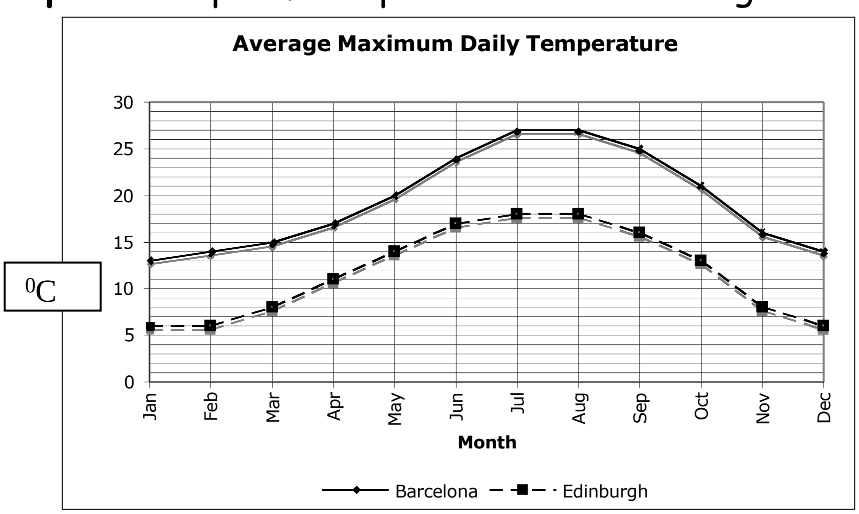
Line graphs consist of a series of points which are plotted, then joined by a line. The trend of a graph is a general

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

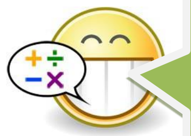


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



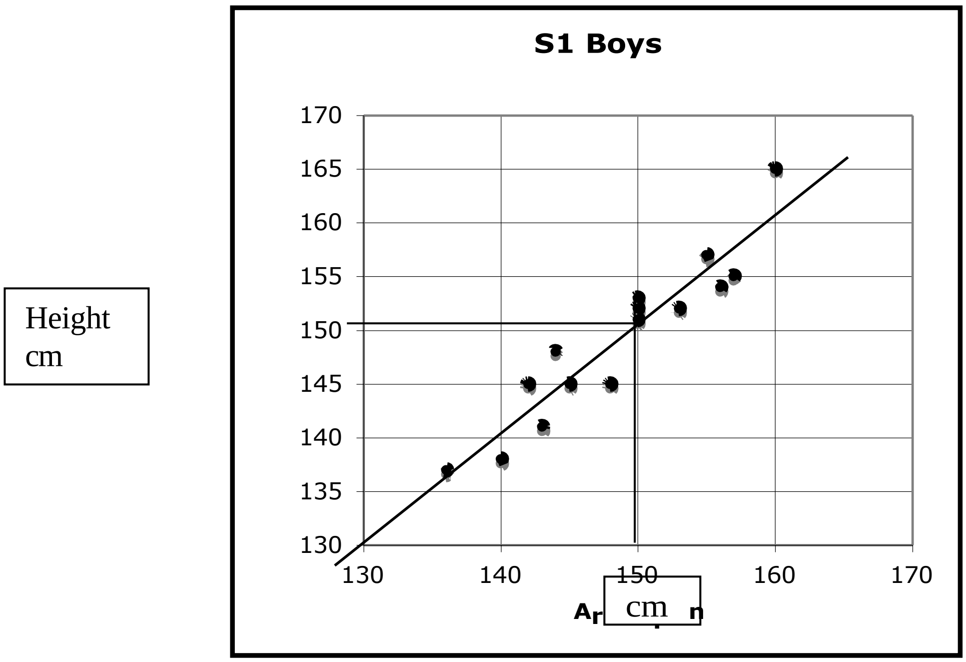
Scatter Graphs



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

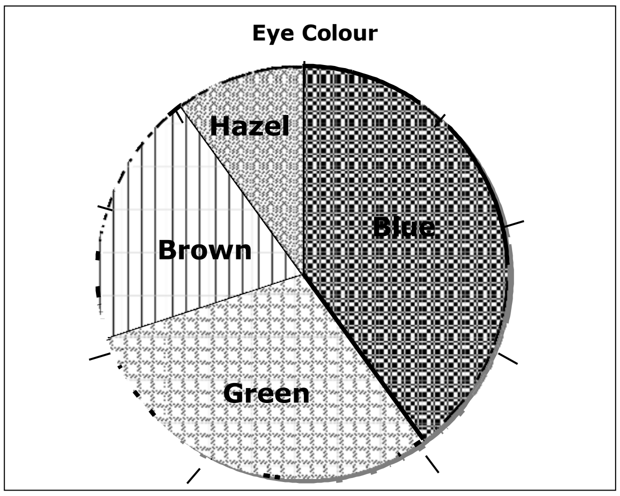


The graph shows a general trend, that **as the arm span increases, so does the height. This graph shows a <u>positive correlation.</u>**
The line drawn is called the **<u>line of best fit</u>**. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.
Note that **<u>in some subjects</u>**, axes may need to start from zero.

Pie Charts

A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.

so the number of pupils with brown eyes

$= \frac{72}{360} \times 30 = 6$ pupils.

If finding all of the values, you can check your answers - the total should be 30 pupils.

Drawing Pie Charts

On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example: In an essay, the number of marks gained on an assignment IS 80 . This is split into Q1, Q2 etc. Draw a pie chart to illustrate the information.

Section of Paper	Number of people
Section 1	28
Section 2	24
Section 3	10
Section 4	12
Spelling Punctuation and grammar	6

Total Marks = 80

$$= \frac{28}{80} = \frac{28}{80} \times 360^\circ = 126^\circ$$

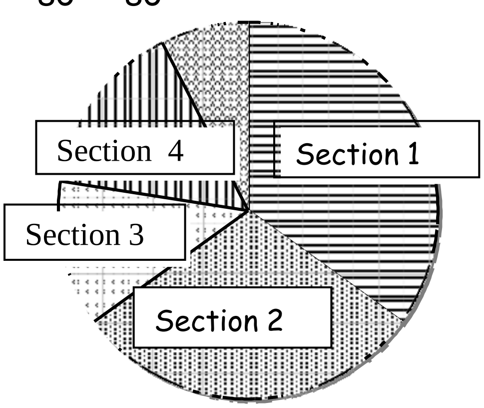
$$= \frac{24}{80} = \frac{24}{80} \times 360^\circ = 108^\circ$$

$$= \frac{10}{80} = \frac{10}{80} \times 360^\circ = 45^\circ$$

$$= \frac{12}{80} = \frac{12}{80} \times 360^\circ = 54^\circ$$

$$= \frac{6}{80} = \frac{6}{80} \times 360^\circ = 27^\circ$$

Check that the total = 360°

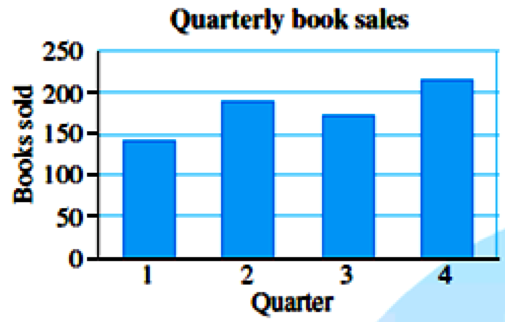


Overall Types of graphs:

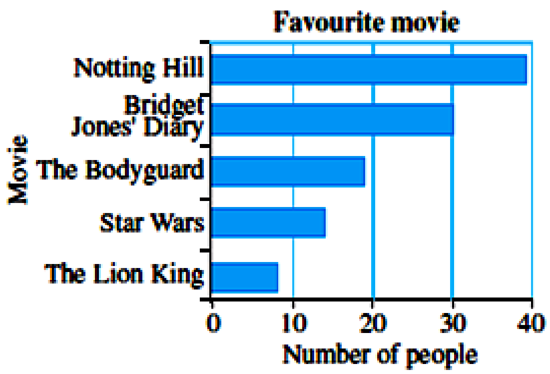
Column graph

In a column graph:

- the data is represented by a number of vertical columns
- the data is shown on the horizontal axis
- the number of times each piece of data occurs is shown on the vertical axis
- multiple columns and stacked columns can be used to compare several data sets directly.



Bar graph



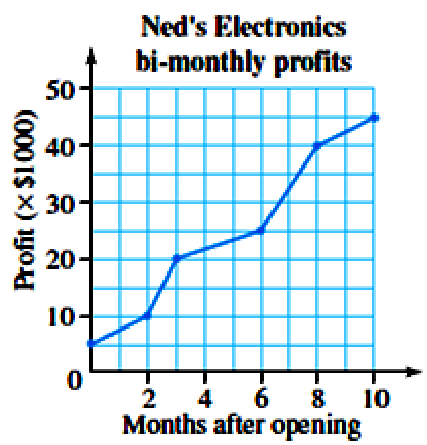
In a bar graph:

- the data is represented by a number of horizontal bars
- the data is shown on the vertical axis
- the number of times each piece of data occurs is shown on the horizontal axis
- multiple bars and stacked bars can be used to compare several data sets directly.

Line graph

In a line graph:

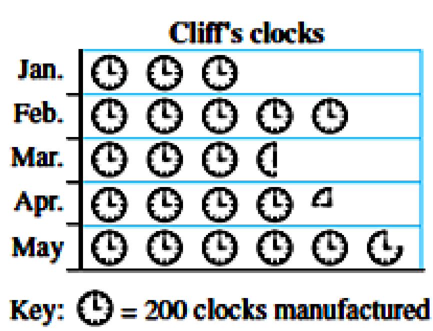
- a number of points are plotted then joined by lines or curves
- trends or relationships between two variables can be shown
- the data are usually obtained by measuring rather than counting, e.g. mass, weight or temperature
- it may be appropriate to determine values between or beyond those that were plotted.



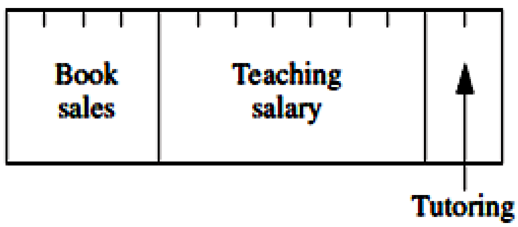
Picture graph

In a picture graph:

- easily recognisable pictures or symbols are used to represent the data
- a key is included to explain the value of the symbol
- accuracy and detail are often sacrificed in favour of visual appeal.



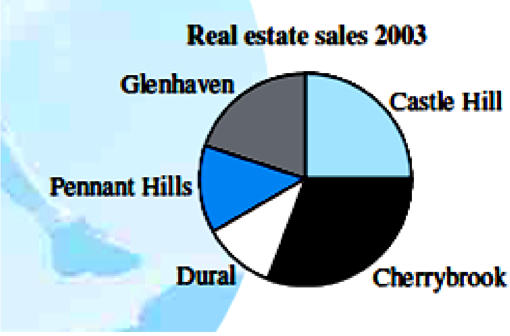
Divided bar graph



In a divided bar graph:

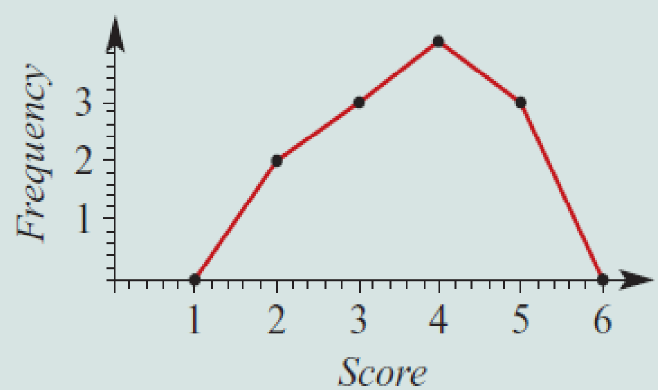
- a rectangle or bar is divided into smaller rectangles or sections
- the length of each section is in proportion to the value of the data that it represents
- the value of the data in each section is found by measuring the length of the section then comparing it to the length of the whole rectangle
- a scale may sometimes be given.

Sector graph



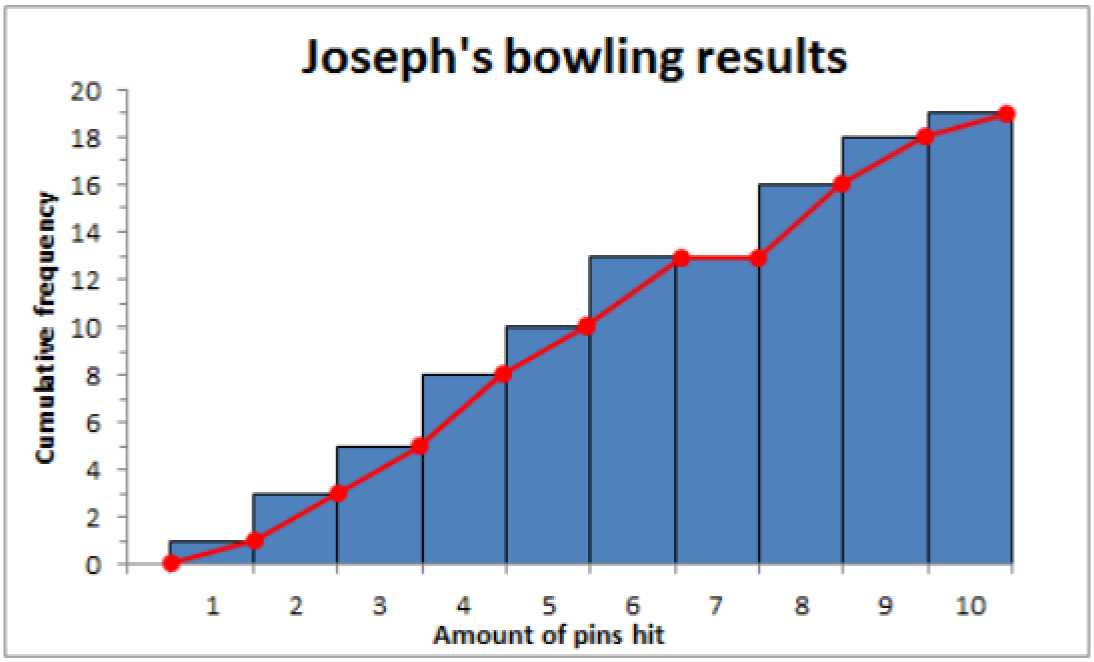
In a sector graph or pie chart:

- a circle is divided into sectors
- the angle at the centre of each sector is in proportion to the value of the data it represents
- the value of the data in each sector is found by dividing the angle at the centre of the sector by 360° then multiplying that fraction by the total value of the data.

Frequency Histogram	Frequency Polygon
Frequency histogram is a column graph of a frequency table.	Frequency polygon is a line graph of a frequency table.
	

Remember:
For the Histogram – start half a unit in, and NO gaps between columns
For the Polygon – join the centre of each column and start at 0 and then join back to 0 at the end

Cumulative Frequency Histogram and Polygon (Ogive)



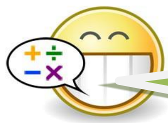
These graphs are slightly different to the NON cumulative ones.

The histogram starts at 0 or 1, not half a unit in.

The ogive joins the bottom left to the top right, and does not go back to 0.

The graph can be used to find medians and percentiles.

Averages



To provide information about a set of data, the average value may be given. There are 3 different types of **average** value – the mean, the median and the mode.

You can remember it by the following rhyme:

> **"HEY DIDDLE DIDDLE, THE MEDIAN'S IN THE MIDDLE, YOU ADD AND DIVIDE FOR THE MEAN. THE MODE IS THE ONE YOU SEE THE MOST, THE RANGE IS THE DIFFERENCE BETWEEN."**

Mean is found by adding all the data together and dividing by the number of values.

Median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode is the value that occurs most often.

Range is the range of a set of data is a measure of spread. = Highest value – Lowest value

Example The temperature each day, over 2 weeks is recorded in °C. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

$$\text{Mean} = \frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14}$$

$$= \frac{102}{14} = 7.285\ldots$$

Mean = 7.3°C to 1 decimal place

Ordered values: 4, 5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 10, 10

Median = 7 °C

7 is the most frequent temperature, so **Mode = 7 °C**

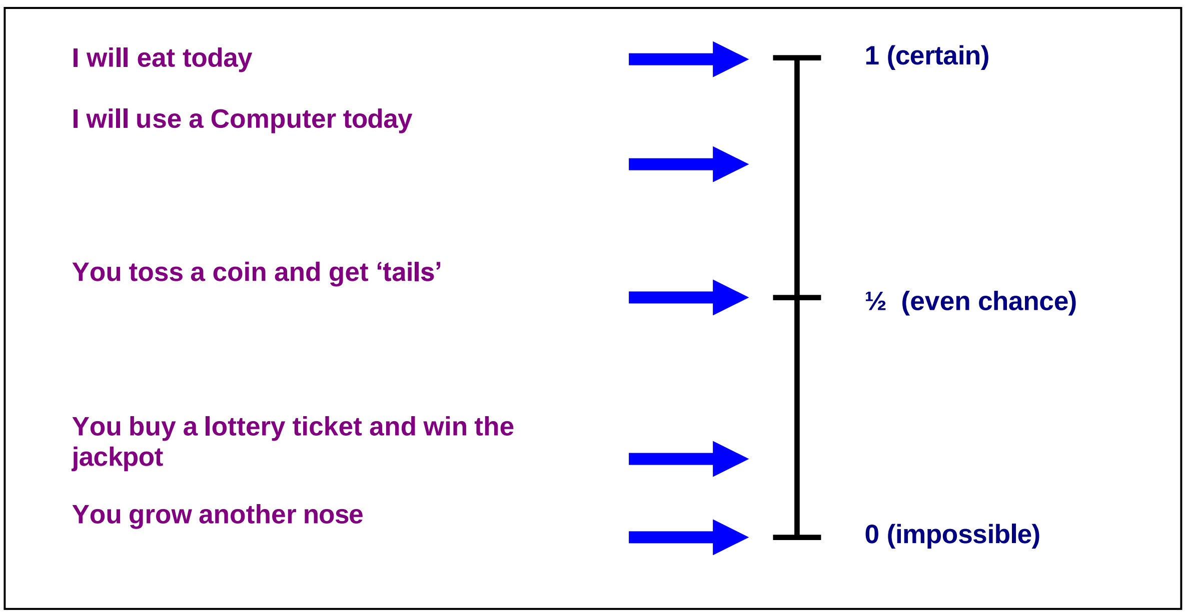
Range = 10 – 4 = 6

Probabilities

We often make judgments as to whether an event will take place, and use words to describe how probable that event is.
For example, we might say that it is likely to be sunny tomorrow, or that it is impossible to find somebody who is more than 3m tall, or it is unlikely I will win the lottery.

The probability scale

In maths we use numbers to describe probabilities. Probabilities can be written as fractions, decimals or percentages. We can also use a probability scale, starting at 0 (impossible) and ending at 1 (certain).



**When we throw a die (plural: dice), there are six possible different outcomes. It can show either 1, 2, 3, 4, 5 or 6.
But how many possible ways are there of obtaining an even number? Clearly, here are three: 2, 4 and 6.
We say that the probability of obtaining an even number is 3/6 (= 1/2 or 0.5 or 50%)**

The probability of an outcome = **number of ways the outcome can happen divided by total number of possible outcomes**

Example 1

How many outcomes are there for the following experiments? List all the possible outcomes.

a) Tossing a coin.

There are two possible outcomes (head and tail).

b) Choosing a sweet from a bag containing 1 red, 1 blue, 1 white and 1 black sweet.

There are four possible outcomes (red, blue, white and black).

c) Choosing a day of the week at random.

There are seven possible outcomes (Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday).

Glossary of Terms

a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon). am = After midnight
Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Area	Amount of surface
Average	Mean, Median and Mode
Bar Graph	One of the ways of presenting data in the form of a graph or chart.
Bargain	An item that has been bought at a reduced price which the customer believes to be a good deal.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Cuboid	Rectangular prism - see triangular prism
Cylinder	Circular prism – see triangular prism
Data	A collection of information (may include facts, numbers or measurements).
Deals	Another term for a special offer.
Decimal places	Places to the right of the decimal point. The first number to the right is the first decimal place.
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction).

	Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Discount	The amount of money that the price of an item has been reduced by , the amount taken off the original price.
Division (÷)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.
Line Graph	One of the ways of presenting data in the form of a graph or chart.

Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p32)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BIDMAS (see p9)
Outcome	An event that can happen
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). pm = past midday

Percentage off	The percentage of the original price.
Percentage of	The percentage of the original price that has been taken off.
Perimeter	Distance around the outside edge
Pie Chart	One of the ways of presenting data in the form of a graph or chart.
Possible	All the possible events that can happen
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Prism	3-dimensional shape with the same cross section along its length
Probability	How likely something is
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Regular Price	The original price that an item has been advertised for before a special offer or discount has been.
Remainder	The amount left over when dividing a number.
Sale Price	The new price an item costs after a discount or special offer.
Scatter Graph	One of the ways of presenting data in the form of a graph or chart.

Share	To divide into equal groups.
Significant Figure	The first non-zero figures in a number which give the most information about the size of the number.
Sphere	A 3D Solid circular shape
Stem & Leaf Diagram	Different ways of presenting data in the form of a graph or chart.
Sum	The total of a group of numbers (found by adding).
Table	Different ways of presenting data in the form of a graph or chart.
Timetable	A table showing the times that someone or something is planned to arrive and depart.
Total	The sum of a group of numbers (found by adding).
Triangular Prism	3-dimensional shape with a triangular cross section along its length
Volume	Amount of space inside a shape or the amount of space an object takes up